**CMSC474 HW2**

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**Problem1**

1. (C,C); (D,C); (E,C); (C,D)
2. (D,D)

**Problem2**

1. No, a positive affine transformation indicates u(x)’ = c\*u(x) + d. If we change all of 2s into 3, by setting up a linear system (0,0) = c\*(0,0) + d && (3,x) = c\*(2,x) + d, we have d = 0 and c = 1.5. Therefore in this case all of 1s should be changed to 1.5 to satisfy a positive affine transformation.
2. (A,A); (B,B)

C) It is equivalent to the unmodified version, since they both have the same Nash equilibria, which is (A,A) and (B,B), and hence any rational agents would do the same choice in this new game.

**Problem3**

1. S->B->A->D

Proof: Suppose that c1 drivers choose the path S->B->D and c2 drivers choose the path S->A->D, then we have

uS->B->D: (1000-c2)/25 + 50 = 90-c1/25

uS->A->D: (1000-c1)/25 + 50 = 90-c2/25

uS->B->A->D: (1000-c2)/25 + (1000-c1)/25 + 5 = 85 - c1/25 - c2/25

Obviously uSBAD is less than any others, therefore for any ai in agents 1 to 1000, if the strategy profiles for a-i are all given, then no matter what c1 and c2 is. The agent ai would only choose SBAD. Hence by 1) There are only three routes to get to D 2)SBAD is the Nash equilibria among those three routes, we have SBAD is the only Nash equilibria in this case.

B)

Three routes in total:

Strategy1: S->A->D

Strategy2: S->B->D

Strategy3: S->B->A->D

Suppose c choose SBD. Then

uSAD = 50 + (1000-c)/25 = 90 - c/25

uSBD = 50 + c/25

uSBAD = 50 + (1000-c)/25 = 90 - c/25.

Now, for every agent in SBD, if one is in a Nash equilibrium, then one can not do better by unilaterally changing his/her strategy if 50 + c/25 <= 90 - c/25 ==> c <= 500. On the other hand, for any agents that are not in SBD, if they are also in a Nash equilibrium then 50+c/25 >= 90-c/25 by definition. Therefore, the Nash equilibrium in this case is u(s, 500, 500-s), where s is any integer in [0, 500].

**Problem4**

For this problem,

the utilitarian function = avg. Expected cost

The egalitarian function = max. Expected cost

Social optimum s\*= arg min c(s)

* 1. The only Nash equilibrium in this case is when all agents choose the path S->B->A->D. Therefore, the utilitarian cost equals to the egalitarian cost, which is 1000/25+5+1000/25 = 85. While the social optimum cost is when we have 500 agents choosing S->A->D and the others choosing S->B->D, which is 50+500/25 = 70. The price of anarchy are both equal to 85/70=17/14
  2. Though the Nash equilibrium in this case can be any combinations of u(s, 500, 500-s), changes to the value of s would not make any difference to the cost function. Therefore, utilitarian cost = egalitarian cost = 50 + 500/25 = 70. The social optimum for both two cases can also be expressed by u(s, 500, 500-s). Price of anarchy in this case equals to 1.